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CLASS OF ALMOST UNBIASED RATIO AND PRODUCT ESTIMATORS IN SYSTEMATIC SAMPLING

K. S. KUSHWAHA and H. P. SINGH
J. N. K. V. V., Jabalpur 482 084

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SUMMARY

In this paper, a general class of almost unbiased ratio and product type estimators are proposed for estimating population mean $\overline{Y}$ of the study characteristic y employing jack knife technique introduced by Quenouille [2], [3]. Explicit expression for the variance of class of estimators is obtained to the first order of approximation. Minimum variance unbiased estimators (optimum estimators) in the class are also identified. The study is confined to systematic sampling.

Keywords : Ratio and product estimator, Jack knife technique, systematic sampling, optimum estimator.

Introduction

The classical ratio and product estimators under systematic sampling scheme were proposed by Swain [6], and Shukla [4] respectively and their properties were studied. In general, the estimators proposed by Swain [6] and Shukla [4] are biased. The bias is small when sample size n is large. However, unusual situations with a large coefficient of variation of auxiliary characteristic x may exist and then the possibility of large bias can arise. It is therefore desirable to either reduce the bias or completely eliminate it. The procedure introduced by Quenouille [2], [3] and later termed as Jack knife by Tukey [7] has been profitably employed in several estimation and testing problems. In the present study, we have used Jack knife technique to get rid of bias in the usual ratio and product estimators and proposed a general class of ratio and product type estimators in systematic sampling. The variance formula for the class of estimators is derived under large sample approximation.

2. Class of Ratio Type Estimators

Suppose the population consists of N units $U = (U_1, U_2, \ldots, U_N)$ numbered from 1 to N in some order. Unless mentioned otherwise, assume $N = nk$ where n and k are positive integers Thus, there will be k samples (clusters) each of size n. We select one sample at random out of k samples and observe both the characteristics x and y for each and every unit selected in the sample. Let $(x_{ij}, y_{ij}, i = 1, 2, \ldots, k, j = 1, 2, \ldots, n)$ denote the value of jth unit in the ith sample. The systematic sample means are

$$\bar{y} = \frac{1}{n} \sum_{j=1}^{n} y_{ij}, \quad \bar{x} = \frac{1}{n} \sum_{j}^{n} x_{ij}, \quad (i = 1, 2, \ldots, k)$$

where sample means $(\bar{y}, \bar{x})$ are unbiased estimators of the population means $(\bar{Y}, \bar{X})$ respectively.

In forming a ratio estimator $\bar{y}_r$ for $\bar{Y}$ based on a systematic sample $(y_{ij}, x_{ij}, j = 1, 2, \ldots, n)$ of size n, we have

$$\bar{y}_r = \frac{\bar{y}}{\bar{x}} \bar{X} \tag{2.1}$$

Swain [6] showed that $\bar{y}_r$ is biased for $\bar{Y}$. To reduce the bias of $\bar{y}_r$, use Jack knife technique due to Quenouille [3]. Here, take $n = gm$ and split the selected systematic sample of size n into g subsamples each of size m in a systematic manner, as this avoides the need for selecting the sample in the form of subsamples of smaller size, thereby retaining the efficiency generally obtained by taking a large systematic sample. Let $\bar{y}_t, \bar{x}_t$ $(t = 1, 2, \ldots, g)$ be unbiased estimators of $(\bar{Y}, \bar{X})$ respectively based on systematic subsamples each of size m. Consider the following ratio type estimators based on subsamples.

$$\bar{y}_{r.} = \frac{1}{g} \sum_{t=1}^{g} \frac{\bar{y}_t}{\bar{x}_t} \bar{X} \tag{2.2}$$

The expressions for biases of $\bar{y}_r$ and $\bar{y}_{r.}$ to the terms of order $O(n^{-1})$ can be easily obtained as

$$\beta_1 (\bar{y}_r) = \frac{\bar{Y}}{n} \left\{ 1 + (n-1) \rho_w \right\} (1 - k^*) C_x^2, \quad \text{and} \tag{2.3}$$

$$\beta_1 (\bar{y}_{r.}) = \frac{\bar{Y}}{n} \left\{ g + (n-g) \rho_w \right\} (1 - k^*) C_x^2$$

where

$$\beta_1 (\bar{y}_r) = \frac{\bar{Y}}{n} \left[\frac{V(\bar{x})}{\bar{X}^2} - \frac{\text{Cov}(\bar{y}, \bar{x})}{\bar{Y}\,\bar{X}} \right]$$

$$V(\bar{x}) = \frac{nk-1}{nk} \frac{S_x^2}{n} \left\{ 1 + (n-1) \rho_{xx} \right\} \approx \frac{S_x^2}{n} \left\{ 1 + (n-1) \rho_w \right\}$$

$$\text{Cov}(\bar{y}, \bar{x}) = \frac{nk-1}{nk} \cdot \frac{\rho_{yx} S_x S_y}{n} \sqrt{1 + (n-1) \rho_{xx}} \sqrt{1 + (n-1) \rho_{yy}}$$

$$\simeq \frac{\rho_{yx}.\, S_x.\, S_y}{n} \left\{ 1 + (n-1) \rho_w \right\}, \text{ (since } (nk-1)/nk \approx 1)$$

$$\rho_{xx} = \frac{E(x_{ij} - \bar{X})(x_{ij'} - \bar{X})}{E(x_{ij} - \bar{X})^2} = \rho_w \text{ (say)}$$

$$\rho_{yy} = \frac{E(y_{ij} - \bar{Y})(y_{ij'} - \bar{Y})}{E(y_{ij} - \bar{Y})^2} = \rho_w \text{ (say)}$$

$$\rho_{yx} = \frac{E(y_{ij} - \bar{Y})(x_{ij} - \bar{X})}{\sqrt{E(y_{ij} - \bar{Y})^2 E(x_{ij} - \bar{X})^2}}$$

$$k^* = \rho_{yx} \frac{C_y}{C_x}$$

where ρ_{yx} is the correlation coefficient between two characteristics x and y in the population, and ρ_w is the intraclass correlation coefficient. For both the characteristics y and x, ρ_w has been assumed to be the same and known (see [1]), C_x, C_y are the coefficient of variation for x and y respectively.

A weighted class of estimators for $\bar{Y}$ is proposed as

$$T_r = \alpha_1 \bar{y} + \alpha_2 \bar{y}_{r\cdot} + \alpha_3 \bar{y}_r, \sum_{l=1}^{3} \alpha_l = 1, \tag{2.4}$$

Where α_l ($l = 1, 2, 3$) are suitably chosen weights given to different estimators. We have the following theorem.

THEOREM 2.1 : *The weighted class of estimators T_r in* (2.4) *for population mean $\bar{Y}$ is unbiased if and anly if*

$$h\,\alpha_3 + \alpha_2 = 0$$

for
$$h = \frac{g + (n - g)\,\rho_w}{1 + (n - 1)\,\rho_w} \tag{2.5}$$

which can be proved easily.

If we take $\alpha_1 = \alpha$, $\alpha_2 = \beta$ and $\alpha_3 = (1 - \alpha - \beta)$, the unbiasedness condition (2.5) reduces to

$$\beta = -\left(\frac{1-\alpha}{h-1}\right)$$

where α and β are constants to be chosen suitably. Thus, obtain a general class of almost unbiased ratio type estimators as

$$\text{Trg} = \alpha\bar{y} - \left(\frac{1-\alpha}{h-1}\right)\bar{y}_{r\cdot} + \left(\frac{1-\alpha}{h-1}\right) h\,\bar{y}_r \tag{2 6}$$

3. Properties of the Class

The variance of the proposed class of estimators Tr in (2.4) is given by

$$V(\text{Tr}) = \alpha_1^2\, v(\bar{y}) + \alpha_2^2\, v(\bar{y}_{r\cdot}) + \alpha_3^2\, v(\bar{y}_r) + 2\,\alpha_1\alpha_2 \operatorname{cov}(\bar{y}, \bar{y}_{r\cdot}) + 2\,\alpha_2\alpha_3 \operatorname{cov}(\bar{y}_{r\cdot}, \bar{y}_r) + 2\,\alpha_3\alpha_1 \operatorname{cov}(\bar{y}_r, \bar{y}) \tag{3.1}$$

To the terms of order $O(n^{-1})$, the variance and covariance expressions for various estimators in (3.1) are cited and proved in the lemma (3.1)

LEMMA (3.1)

$$V(\bar{y}) = \frac{\bar{Y}^2}{n}\left\{1 + (n-1)\rho_w\right\} C_y^2$$

$$V(\bar{y}_{r\cdot}) = V(\bar{y}_r) = \text{cov}(\bar{y}_{r\cdot}, \bar{y}_r) \tag{3.2}$$

$$= \frac{\bar{Y}^2}{n}\left\{1 + (n-1)\rho_w\right\}\left\{C_y^2 + (1 - 2k^*)C_x^2\right\}$$

$$\text{Cov}(\bar{y}, \bar{y}_{r\cdot}) = \text{cov}(\bar{y}, \bar{y}_r) = \frac{\bar{Y}^2}{n}\left\{1 + (n-1)\rho_w\right\}(C_y^2 - k^* C_x^2)$$

Proof: Following Sukhatme *et al.* [5, pp. 162-164],

$$V(\bar{y}_{r\cdot}) = V(\bar{y}_r) = \text{cov}(\bar{y}_{r\cdot}, y_r)$$

and $\text{cov}(\bar{y}, \bar{y}_{r\cdot}) = \text{cov}(\bar{y}, \bar{y}_r)$

To find the expressions for $\text{cov}(\bar{y}_{r\cdot}, \bar{y}_r)$ and $\text{cov}(\bar{y}, \bar{y}_r)$ under large sample approximation, let

$$\bar{y} = \bar{Y}(1 + e_0) \Rightarrow e_0 = \frac{\bar{y} - \bar{Y}}{\bar{Y}}$$

$$\bar{x} = \bar{X}(1 + e_1) \Rightarrow e_1 = \frac{\bar{x} - \bar{X}}{\bar{X}}$$

We have $(\bar{y}_r - \bar{Y}) = \bar{Y}[(1 + e_0)(1 + e_1)^{-1} - 1]$ (3.3)

Again, let

$$\bar{y}_t = \bar{Y}(1 + e_0')$$

$$\bar{x}_t = \bar{X}(1 + e_1')$$

We have

$$(\bar{y}_{rt} - \bar{Y}) = \bar{Y}[(1 + e_0')(1 + e_1')^{-1} - 1]$$

$$\Rightarrow (\bar{y}_{r\cdot} - \bar{Y}) = \bar{Y}[(1 + e_0')(1 + e_1') - 1]$$

Using (3.3) and (3.4) together, we have

$$\begin{aligned}\text{cov}(\bar{y}_r, \bar{y}_{r\cdot}) &= E(\bar{y}_r - \bar{Y})(\bar{y}_{r\cdot} - \bar{Y})\\ &= \bar{Y}^2 E[(1 + e_0)(1 + e_1)^{-1} - 1][(1 + e_0')(1 + e_1')^{-1} - 1]\end{aligned} \tag{3.5}$$

where $|e_1| < 1$, and $|e_1'| < 1$, so that the functions $(1 + e_1)^{-1}$ and $(1 + e_1')^{-1}$ can be expanded and in the expansion of (3.5), considering the terms in e's only upto power two, we have

$$\begin{aligned}\text{Cov}(\bar{y}_r, \bar{y}_{r\cdot}) &= \bar{Y}^2 E[e_0 e_0' - e_0 e_1' - e_1 e_0' + e_1 e_1']\\ &= \bar{Y}^2 E_1 [e_0 E_2 (e_0'/t) - e_0 E_2 (e_1'/t) - e_1 E_2 (e'/t)\\ &\quad + e_1 E_2 (e_1'/t)]\end{aligned} \tag{3.5a}$$

where $E_2(e_0'/t) = \dfrac{E_2(\bar{y}_t/t) - \bar{Y}}{\bar{Y}}$ is the conditional expectation for a given tth $(t = 1, 2, \ldots, g)$, split and E_1 is the expectation on ith $(i = 1, 2, \ldots, k)$ systematic sample of size n. Here, it is to be noted that y_t is a systematic sample mean of size m drawn from a population of size n and hence $E_2(\bar{y}t/t) = \bar{y}$. Thus, $E_2(e_0'/t) = e_0$ and similarly $E_2(e_1'/t) = e_1$.

Thus, from (3.5a), we have

$$\begin{aligned}\text{Cov}(\bar{y}_r, \bar{y}_{r\cdot}) &= \bar{Y}^2 E_1 [e_0^2 + e_1^2 - 2 e_0 e_1]\\ &= \bar{Y}^2 \left[\frac{V(\bar{y})}{\bar{Y}^2} + \frac{V(\bar{x})}{\bar{X}} - 2\frac{\text{Cov}(\bar{y}, \bar{x})}{\bar{Y}\bar{X}}\right]\\ &= \frac{\bar{Y}^2}{n}\left\{1 + (n-1)\rho_w\right\}\left[C_y^2 + C_x^2 - 2 C_x C_y\right]\\ &= \frac{\bar{Y}^2}{n}\left\{1 + (n-1)\rho_w\right\}\left[C_y^2 + C_x^2(1 - 2k^*)\right]\end{aligned} \tag{3.6}$$

Similarly for cov $(\bar{y}, \bar{y}_r)$ we have

$$\text{Cov}(\bar{y}, \bar{y}_r) = E(\bar{y} - \bar{Y})(\bar{y}_r - \bar{Y})$$

$$= \bar{Y}^2 E(e_0^2 - e_0 e_1)$$

$$= \bar{Y}_2 \left[\frac{V(\bar{y})}{\bar{Y}_2} - \frac{\text{Cov}(\bar{y}, \bar{x})}{\bar{Y}\bar{X}} \right]$$

$$= \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1)\rho_w \right\} \left(C_y^2 - k^* C_x^2 \right) \tag{3.7}$$

Substituting the results (3.2) and $\alpha^2 = \alpha$, $\alpha_1 = \beta$ and $\alpha_3 = (1 - \alpha - \beta)$ in (3.1), we obtain the variance formula for the class of estimators T_{rg} given as

$$V(T_{rg}) = \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1)\rho_w \right\} \left(C_y^2 + (1-\alpha)\{(1-\alpha) - 2k^*\} C_x^2 \right) \tag{3.8}$$

The variance of T_{rg} in (3.8) will be minimum for

$$\alpha = (1 - k^*) = \alpha^* \text{ (Say)} \tag{3.9}$$

Thus, the minimum variance of T_{rg} is given by

$$\text{Minm } V(T_{rg}) = \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1)\rho_w \right\} (C_y^2 - k^{*2} C_x^2) \tag{3.10}$$

which is equivalent to the approximate variance of usual biased linear regression estimator $\bar{y}_{1r}$ in systematic sampling given as

$$\bar{y}_{1r} = \bar{y} + b_{yx}(\bar{X} - \bar{x}) \tag{3.11}$$

where b_{yx} to the sample regression coefficient of y on x. Substituting $\alpha_1 = \alpha^* = (1 - k^*)$, $\alpha_2 = \beta^* = -k^*/(h-1)$ and $\alpha_3 = (1 - \alpha^* - \beta^*) = (k^*h)/(h-1)$ in (2.4), we obtain an optimum estimator in the class T_{rg} given as

$$\text{Tro} = (1 - k^*)\bar{y} - \left(\frac{k^*}{h-1} \right) \bar{y}_{r'} + \left(\frac{k^*h}{h-1} \right) \bar{y}_r \tag{3.12}$$

with variance (Minm $V(T_{rg})$ given in (3.10).

It is to be pointed out that the class T_{rg} (2.6) of ratio type estimators would be more efficient than the conventional unbiased estimator $\bar{y}$ and the ratio estimator $\hat{y}_r$ due to Swain [6] according if—

$$\left.\begin{array}{ll} \text{either} & 0 < \alpha < (1 - 2k^*) \\ \text{or} & (1 - 2k^*) < \alpha < 1 \end{array}\right\} \quad (3.13)$$

and

$$\left.\begin{array}{ll} \text{either} & 0 < \alpha < 2(1 - k^*) \\ \text{or} & 2(1 - k^*) < \alpha < 0 \end{array}\right\} \quad (3.14)$$

4. Class of Product Type Estimators

In this section, we propose a general class of estimators altenative to ratio type estimators. In forming a product estimator $\hat{y}_p$ for population mean $\bar{Y}$ based on a systematic sample $(y_{ij}), x_{ij}, j = 1, 2, \ldots, n)$ of size n, we have

$$\overline{y_p} = \frac{\hat{y}\,\bar{x}}{\bar{X}} \quad (4.1)$$

Shukla [4] showed that $\bar{y}_p$ is biased for $\bar{Y}$. We use the technique adopted in section (2) to get a general class of unbiased product type estimators. We consider the following product type estimators based on g subsamples.

$$\overline{y_{p.}} = \frac{1}{g}\sum_{t}^{g} \frac{\hat{y}_t\,\bar{x}_t}{\bar{X}} \quad (4.2)$$

The expressions for the biases of $\overline{y_p}$ and $\overline{y_{p.}}$ to the terms of order $O(n^{-1})$ can be easily obtained as

$$\beta_1(\overline{y_p}) = \frac{\bar{Y}}{n}\left\{1 + (n-1)\,\rho_w\right\} k^* C_x^2 \quad (4.3)$$

and $\beta_1(\hat{y}_{p.}) = \frac{\bar{Y}}{n}\left\{g + (n-g)\,\rho_w\right\} k^* C_x^2$

We propose a weighted class of estimators for $\bar{Y}$ as

$$T_p = w_1 \bar{y} + w_2 \bar{y}_{p.} + w_3 \bar{y}_p, \sum_{l=1}^{3} w_l = 1 \tag{4.4}$$

where w_l ($l = 1, 2, 3$) are suitably chosen weights given to different estimators. We have the following theorem.

THEOREM 4.1 : *The weighted class of estimators T_p in* (4.4) *for population mean $\bar{Y}$ is unbiased if and only if*

$$h\, w_2 + w_3 = 0$$

$$\text{for } h = \frac{g + (n - g)\,\rho_w}{1 + (n - 1)\,\rho_w} \tag{4.5}$$

which can be proved easily.

If we take $w_1 = w$, $w_2 = \gamma$ and $w_3 = (1 - w - \gamma)$, the condition (4.5) reduces to

$$\gamma = -\frac{(1 - w)}{(h - 1)}$$

where w and γ are constants to be chosen suitably. Thus, we obtain a general class of unbiased product type estimators

$$T_{pg} = w\bar{y} - \left(\frac{1 - w}{h - 1}\right)\bar{y}_{p.} + \left(\frac{1 - w}{h - 1}\right) h\, \bar{y}_p \tag{4.6}$$

5. Properties of the Class

The variance of the proposed class of estimators T_p in (4.4) is given by

$$V(T_p) = w^2\, v(\bar{y}) + w_2^2\, v(\bar{y}_{p.}) + w^2\, v(\bar{y}_p) + 2\, w_1 w_2 \operatorname{cov}(\bar{y}, \bar{y}_{p.})$$

$$+ 2\, w_2 w_3 \operatorname{cov}(\bar{y}_{p.}, \bar{y}_p) + 2\, w_3 w_1 \operatorname{cov}(\bar{y}_p, \bar{y}) \tag{5.1}$$

To the terms of order $O(n^{-1})$, the variance and covariance expressions for various estimators in (5.1) are cited in lemma (5.1).

LEMMA (5.1) :

$$V(\bar{y}) = \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1) \rho_w \right\} C_y^2$$

$$V(\bar{y}_{p\cdot}) = V(\bar{y}_p) = \text{Cov}(\bar{y}_{p\cdot}, \bar{y}_p) \tag{5.2}$$

$$= \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1) \rho_w \right\} \left\{ C_y^2 + (1 + 2k^*) C_x^2 \right\}$$

$$\text{Cov}(\bar{y}, \bar{y}_p) = \text{Cov}(\bar{y}, \bar{y}_p) = \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1) \rho_w \right\} (C_y^2 + k^* C_x^2)$$

which can be proved easily as in Section (3).

Substituting the results (5.2) and $w_1 = w$, $w_2 = \gamma$ and $w_3 = (1-w-\gamma)$ in (5.1), we obtain the variance formula for the class of estimators T_{pg} given as

$$V(T_{pg}) = \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1) \rho_w \right\} \left[C_y^2 + (1-w)\{(1-w) + 2k^*\} C_x^2 \right] \tag{5.3}$$

The variance of T_{pg} in (5.3) will be minimum for

$$w = 1 + k^* = w^* \quad \text{(say)} \tag{5.4}$$

Thus the minimum variance of T_{pg} is given by

$$\text{Minm } V(T_{pg}) = \frac{\bar{Y}^2}{n} \left\{ 1 + (n-1) \rho_w \right\} (C_y^2 - k^{*2} C_x^2) \tag{5.5}$$

which is equivalent to the approximate variance of usual biased linear regression estimator $\bar{y}_{lr}$ in systematic sampling given as

$$\bar{y}_{lr} = \bar{y} + b_{yx} (\bar{X} - \bar{x}) \tag{5.6}$$

Substituting $w_1 = w^* = (1 + k^*)$, $w_2 = \gamma^* = \dfrac{k^*}{h-1}$

and $w_3 = (1 - w^* - \gamma^*) = -\frac{k^*h}{h-1}$ in (4.4), we obtain an optimum estimator in the class T_{pg} given as

$$T_{po} = (1 + k^*)\,\bar{y} + \left(\frac{k^*}{h-1}\right)\bar{y}_{p\cdot} - \left(\frac{k^*h}{h-1}\right)\bar{y}_p \qquad (5.7)$$

with variance (Minm $V(T_{pg})$ given in (5.5).

It is to be pointed out that the class T_{pg} (4.6) of product type estimators would be more efficient than the conventional unbiased estimator $\bar{y}$ and product estimator $\bar{y}_p$ due to Shukla [4] according if

$$\left.\begin{array}{ll} \text{either} & 1 < w < (1 + 2k^*) \\ \text{or} & (1 + 2k^*) < w < 1 \end{array}\right\} \qquad (5.8)$$

and

$$\left.\begin{array}{ll} \text{either} & 0 < w < 2(1 + k^*) \\ \text{or} & 2(1 + k^*) < w < 0 \end{array}\right\} \qquad (5.9)$$

6. Effect of Sample Sizes on Bias and Sampling Variance of $\bar{y}_r$

To see the effect of different sample sizes on the approximate relative bias and variance of ratio estimator $\bar{y}_r$, the data on volume of timber of 176 forest strips given in ([1], pp. 131-132) have been considered. The relative bias and variance of ratio estimator $\bar{y}_r$ based on systematic sample of sizes 2, 4, 8, 16 and 22 strips by enumerating all possible systematic samples after arranging the data in ascending order of strip length and the value of ρ_w, the intraclass correlation, are shown in Table 6.1. For the systematic sampling to be efficient, the units within the same systematic sample should be as heterogeneous as possible with respect to the characteristic under consideration. As the volume (y) of timber is expected to be related to the strip length (x), the arrangement according to this is likely to be approximately similar to the arrangement according to the volume of timber, the study variable y.

The intraclass correlation ρ_w is calculated as

$$\rho_w = 1 - \frac{n}{n-1}\,\frac{\sigma_w^2}{\sigma^2} \qquad (6.2)$$

where σ^2 and σ_w^2 are the population and within sample variances respectively given as

$$\sigma^2 = \frac{1}{nk} \sum_{1}^{k} \sum_{j}^{n} (x_{ij} - \bar{X})^2$$

$$\sigma_w^2 = \frac{1}{k} \sum_{i=1}^{k} \sigma_{wi}^2 = \frac{1}{k} \sum_{1}^{k} \left(\frac{1}{n} \sum_{j}^{n} (x_{ij} - \bar{x}_i)^2 \right)$$

The intraclass correlation ρ_w generally varies with the sample size and arrangement of units in the population. Summarised data is as follows

$N = 176$

$\bar{Y} = 282.6136$ $\quad C_y^2 = 0.3036$ $\quad S_{xy} = 309.8317$

$\bar{X} = 6.9943$ $\quad C_x^2 = 0.1791$ $\quad \rho_{yx} = 0.6722$

$k^* = 0.8752$

TABLE 6.1 — RELATIVE BIAS AND VARIANCE

Sample size	ρ_w	$R.B. = Bias(\bar{y}_r)/\bar{Y}$	$R.V. = V(\bar{y}_r)/\bar{Y}^2$
n			
2	–0.5119	0.00545	0.04128
4	–0.1510	0.00305	0.02312
8	–0.1106	0.00063	0.00477
16	–0.0522	0.00030	0.00228
22	–0.0435	0.00008	0.00059

Table 6.1 shows that the relative bias and sampling variance (mean square error) of ratio estimator Y_r decreases with increase in n according to a known function of N, n, S_y^2 and S_x^2. By comparing the relative variances of $\bar{y}_r$ for different sample sizes, it is found that the arrange-

ment of units in the order of strip length has been of considerable help in reducing the relative sampling variance except possible for large sample sizes. Another point needing attention is that the relative variance tends to be substantial for larger sample sizes even if intraclass correlation in such cases turns out to be small in magnitude, provided of course it is negative.

Remarks (I) : The value of X is known but the value of $k = \rho_{yx} \dfrac{C_y}{C_x}$ is rarely known. Also $k^* = \beta_{yx} R^{-1}$, $R = \dfrac{\bar{Y}}{\bar{X}}$ and β_{yx} is the slope of population regression line of y on x. An estimate $\bar{y}$ of $\bar{Y}$ is available at the estimation stage and β_{yx} may be assessed with the help of scatter diagram of y against x based on the data from current study and the slope of best fitting line of y on x may be assessed to be used as β_{yx}. Hence the value of k^* may be quite accurately guessed and thus the value of k^* may be used to obtain the feasible estimator.

(II) A serious demerit of systematic sampling is that it is not possible to estimate the sampling variance unbiasedly but using the technique of interpenetrating systematic sampling with independent random start, the sampling variance of the estimator can be estimated unbiasedly.

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